

Metrological Validation of a Digital Model for a CMM including Digital Bias Correction

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Abstract – Virtual experiments and digital twins are virtual representations of simulation processes and can aid in crucial tasks, such as interpreting measurement data or evaluating uncertainties. Therefore, virtual experiments and digital twins are essential components in the digital transformation in metrology. However, it is important that the digital model is properly validated, to make sure its output is reliable. This research shows how to validate a virtual coordinate measurement machine (CMM) by means of reference measurements. The output of the digital model is compared to reference measurements of a ring gauge and its consistency is evaluated. The results show that the output of the digital model for the diameter of the ring gauge is consistent with the reference data. The roundness measurements seem to contain an inherent bias, introduced by the CMM. After correcting for this bias, the output of the digital model for the roundness of the ring gauge is also consistent with the reference data.

I. INTRODUCTION

Virtual experiments and digital twins are detailed software-based simulation tools that can help in estimating the value of a measurand from measurement data or evaluate the uncertainty of such an estimate. See for example in [1], where a framework is established for virtual experiments and digital twins. These tools are seen as a crucial component in the digitalization of metrology, due to their potential uses. Therefore, a lot of research is aimed at developing and using virtual experiments and digital twins, such as in [2] and [3]. However, without properly validating the digital model, the results are not traceable and therefore difficult to use in practice. Typically, these types of digital models can be validated via reference measurements. By performing a reference measurement and processing the measurement data with the virtual experiment, one can test if the outcomes of the digital model are consistent with the known properties of the reference. See for example [4], where a finite element model is validated. In addition, it is also important to know how to deal with inconsistencies in the validation results. To this end a validation study is performed on the virtual coordinate measurement machine (CMM) developed at VSL. The aim of this work is to validate the virtual CMM,

including the values and uncertainties of its parameters. Moreover, a general approach will be studied that is aimed at dealing with potential biases in the measurand estimates, which can improve the measurement results. First, the virtual CMM will be described, including the experimental determination of the virtual CMM's model parameters. Then, the method to evaluate the uncertainty of a new measurement, using the virtual CMM, will be explained, as well as the method to correct for potential biases. Hereafter, the details of the validation measurements are provided, after which the results are discussed. The paper concludes with a discussion of the results.

II. VIRTUAL CMM

The aim of this research is to validate a virtual CMM, which is a virtual representation of a CMM. In other words, a virtual CMM models how error sources influence the measurement data of a CMM, both random and systematic error sources. The measurement data of a CMM includes coordinates that describe the surface of an object. From these coordinates, geometrical features of the object can be derived, i.e. the radius of a circular object. Typically, the geometrical features are estimated via optimization procedures, such as least-squares or minimum zone optimization, see [5]. The remainder of this section describes the errors sources considered in the virtual CMM and how their values and uncertainties are determined experimentally. The order of the sections corresponds to the order in which the different error sources are applied in the virtual CMM.

A. Workpiece surface roughness

Roughness of the surface of the workpiece affects the measured coordinates by the CMM. Roughness can have a significant impact on certain measurands that depend on the coordinates of single data points, such as roundness or flatness, as the roughness effects cannot be averaged out in a single data point. The local roughness is typically unknown, but usually a sufficiently accurate estimate is available for the standard deviation of the roughness. This estimate can, under certain conditions, be determined via CMM measurements. Alternatively, the roughness can be estimated via roughness tables from literature that take the measurement process into account. In the virtual CMM,

the roughness is modelled as random deviations in the direction of the surface normal, where the standard deviation (and therefore the height of the simulated roughness) is determined by a roughness measurement.

B. Workpiece temperature

Changes in the temperature of the workpiece causes the workpiece to expand or shrink. It is therefore important to correct for the workpiece temperature, such that all values for the measurands are reported at the standard reference temperature of 20 °C, as described in [6]. This way, the different measurements can be compared. It is assumed that the workpiece is acclimatized to the room temperature during the waiting time before performing the measurements, such that temperature during the measurement determines the workpiece temperature. Moreover, it is assumed that the temperature of the workpiece is homogenous in both time and space, i.e. there is no temporal and spatial temperature gradient. The uncertainty of the temperature measurement, as well as the uncertainty of the coefficient of thermal expansion of the workpiece, determine the uncertainty contribution of this error source.

C. Kinematic errors

The kinematic errors of a CMM refer to rotations and translations the machine parts can exhibit along the different axes. Considering a 3D CMM this adds up to 18 errors (3 rotation and 3 translation errors per axis). In addition, between each of the axes a squareness error exists, which adds an additional 3 kinematic error sources. The 21 kinematic error sources were characterized (magnitude and uncertainty) by using a calibrated hole plate. This approach is similar to [7].

D. Probe error

The coordinates of the measured object are determined by contact with a probe. The CMM registers the coordinates at the centre of the probe and adds the radius of the probe (in the probing direction) to determine the coordinates of the object surface. However, errors in the radius estimate of the probe tip, as well as any form imperfections of the probe tip will result in errors in the measured coordinates. Furthermore, the electronics and sensors in the active probe head also add to the probe uncertainty. To account for this, the probe error is characterized (value and uncertainty of radius and directional error) by using a calibrated sphere, see also [8].

E. Random noise

In addition to the systematic error sources, there is always random noise present in the data, for example caused by random vibrations. To determine the standard uncertainty of the noise, the CMM measurements of the calibrated artefact were compared to the reference data. It is assumed that the random noise has a high frequency, as

each data point contains random noise in a different direction. The low frequency components of the data therefore constitute the workpiece form deviations and systematic CMM errors. describes the random noise. The low frequency component is determined with the reference data (under the assumption that the noise is negligible in the reference data) and subsequently subtracted from the CMM data to obtain an estimate of the standard deviation of the noise. This analysis was performed for both scanning and touch trigger data, to determine the random noise uncertainty for both measurement techniques.

III. UNCERTAINTY EVALUATION

The use of the virtual CMM is twofold. On one hand, it can be used to correct for known systematic CMM errors affecting the measurement data, to improve the estimate of the measurand. On the other hand, the virtual CMM can be used for uncertainty evaluation. The uncertainties (or rather, probability distributions) of the different error sources can be propagated through the virtual CMM to obtain realistic samples of the measurement data. Subsequently, the uncertainty in the measurands can be determined by applying the fitting routines to the measurement data samples. In this section, the method to evaluate the uncertainty, using the virtual CMM, is described.

Propagation of distributions (PoD) is applied to evaluate the uncertainty in the measurands. To this end, the Monte Carlo method (MC) is employed. First, random samples of the error sources are drawn based on the respective uncertainties that are given by the characterizations. Then, for each random sample, the errors are applied to the measurement data, using the virtual CMM. Finally, the optimization procedure determines the measurands for each sample. This results in MC samples of the measurands, from which the standard uncertainty and other statistics can be derived.

Additionally, a method discussed in [9] was employed. Here, it is argued that potential biases can be recognized by the virtual experiment. The values of the measurands for the input data can be determined by applying the fitting routines. If these estimates differ from the means of the MC samples (which have the measurement data as input), then the errors introduce a bias in the estimates of the measurands. The estimate of this bias is equal to the difference between the estimate of the measurands based on the input data and the mean of the estimates provided by the MC samples. It stands to reason that this bias is also present in the measurement data, assuming that the virtual CMM is a realistic representation of the actual CMM. Therefore, the estimate of the measurands can be improved by subtracting the bias. In contrast to [9], in this paper the approach for bias correction is applied to real measurement data instead of synthetic data.

IV. MEASUREMENTS

In order to validate the virtual CMM and its uncertainty evaluation described in the previous sections, several measurements were performed on a ring gauge. In this validation, the diameter and roundness of the inner ring with a nominal diameter of 171 mm were studied. The roundness (RONt – peak-to-valley roundness deviation) is defined as the maximum deviation from the reference circle minus the minimum deviation from the reference circle, consistent with [5]. The reference circle is described by a maximum inscribed circle fit of the inner ring. Moreover, a low-pass Gaussian filter has been applied to the data before determining the roundness, to reduce the impact of random noise. This is also in line with [5].

First, reference measurements were performed. The diameter of the inner ring was measured with a laser interferometer, based on two points of the ring gauge (at 0° and 180°). In addition, a roundness meter (Mitutoyo Roundtest RA-H5100) measured the full profile of the inner ring. From this profile, the reference roundness of the inner ring was derived. Moreover, the average diameter over the entire ring gauge was derived from the profile. First the average profile deviation was set to zero at 0° and 180° and all other profile deviations were adjusted accordingly. Hereafter, the average deviation of the entire profile with respect to these two points was determined. See Figure 1 for an example of the measured ring gauge profile.

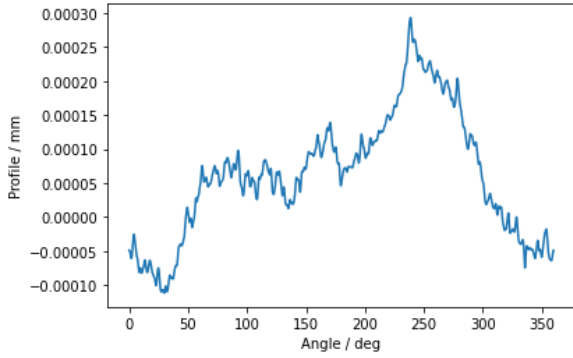


Figure 1: Profile of the ring gauge, as measured by the roundness meter.

Finally, the reference diameter was defined as the distance between the 0° and 180° points of the ring gauge, as measured by the laser interferometer, plus the average deviation from this distance of the entire profile, as measured by the roundness meter. This process is repeated three times: at the top, in the middle and at the bottom of the ring gauge.

The uncertainty of the reference roundness follows from the uncertainty budget of the roundness meter. The uncertainty of the reference diameter is a combination of the uncertainty of the laser interferometer measurement, as well as the uncertainty of a single point of the profile roundness measurement. Note that the uncertainty of the

roundness meter is negligible compared to the uncertainty of the laser interferometer, so in practice the uncertainty of the reference diameter is equal to the uncertainty of the laser interferometer measurement.

In addition to the reference measurement, multiple measurements were performed on the CMM, measuring the calibrated ring gauge. By applying the virtual CMM to the measurement data, the diameter and roundness were determined and their uncertainties were evaluated. Consistency between the results of the reference measurements and the CMM measurements provides the validation of the virtual CMM. This consistency was evaluated with the well-known E_n value, which is given by

$$E_n = \frac{x_{\text{ref}} - x_{\text{meas}}}{k\sqrt{u_{\text{ref}}^2 + u_{\text{meas}}^2}} \quad (1)$$

where x_{ref} and x_{meas} are the estimates for the measurands of the reference and validation measurement, respectively, k is the coverage factor that was set equal to 2 and u_{ref} and u_{meas} are the standard uncertainties of the reference and validation measurement, respectively.

The idea behind the performed measurements is to test the performance limits of the CMM, in order to validate the virtual CMM to the largest extend. First of all, the measurements were repeated at different locations in the measurement volume of the CMM. Typically, measurements are performed in the middle of the CMM, but during this validation study additional measurements were performed at the corners of the CMM, as well as at elevated levels. By measuring at 7 different locations, the entire measurement volume of the CMM was validated. Moreover, the ring gauge was rotated during several measurements. This allows for making a distinction between the effects due to the shape of the ring gauge (which is rotated in these measurements) and the effects due to errors in the CMM (which remain constant between the measurements). The ring gauge was measured in 4 different orientations. Finally, all measurements are repeated twice, one scanning measurement and one touch trigger measurement, at 3 locations of the ring gauge (top, middle, bottom), leading to a total of 66 measurements.

V. RESULTS

In this section, the results of the virtual CMM validation study are discussed. First, the E_n values are discussed to provide a general overview of the validation results. Then, the results for the different measurement locations are compared, as well as the results where the ring gauge is rotated at different angles. Finally, the results after correcting for the bias corrections are discussed.

A. E_n values

The E_n values, calculated according to equation (1), of the diameter and roundness for the different measurements can be found in Figure 2 and Figure 3, respectively. For

the diameter measurements, all E_n values are between the thresholds of -1 and 1. This provides a strong indication that the diameter estimates are consistent with the reference measurements. The diameter seems to be a bit underestimated on average, as most E_n values are above 0. This is potentially due to the fact that the kinematic errors that are not corrected for, but only taken into account in the uncertainty. For the roundness, about 6 % of the measurements have an E_n value below -1. Moreover, all roundness estimates are higher than the reference roundness, as all E_n values are below 0. This is similar to the observations in [9], where it was found that there is a bias in the roundness estimate. Therefore, a correction for this bias, as discussed in section IV, seems appropriate.

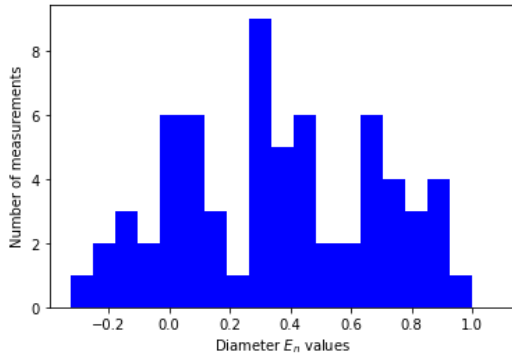


Figure 2: E_n values of the measured diameter.

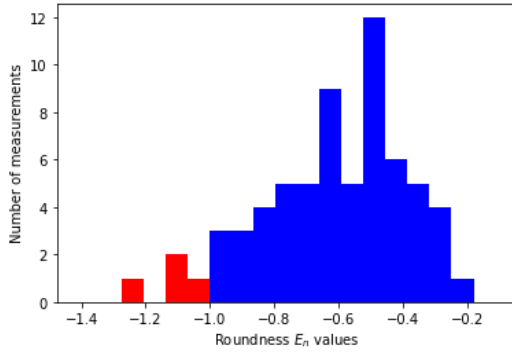


Figure 3: E_n values of the measured roundness.

B. Location dependence

The measurements were repeated in different locations of the measurement volume of the CMM. This way, the measurement location dependency could be studied. In Figure 4 a comparison is made between two different measurement locations. This figure shows the measurement residuals with respect to the reference circle of two different measurements at the centre of the CMM measurement volume, versus residuals in of one measurement in the corner of the measurement volume. It is clear that the measurements performed in the centre of the measurement volume are very comparable, while they

are distinctly different from the measurement performed at the corner of the measurement volume. This means that the (kinematic) errors strongly depend on the measurement location and it is therefore good to take the location dependency into account when applying/correcting the kinematic errors.

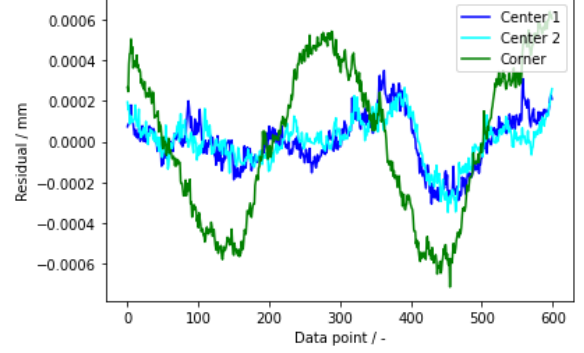


Figure 4: Comparison of residuals with respect to reference circle for measurements performed in the centre of the CMM versus the corner of the CMM.

C. Rotation

In Figure 5 the differences between the different rotation measurements are shown. The residuals with respect to the reference circle are very similar between the different rotation measurements. This shows that the measurement errors in the CMM are more dominant than the form deviations in the ring. Indeed, the measurement uncertainty for a single datapoint is approximately 0.8 micrometre for these measurements. The deviations shown in Figure 5 fall well within this uncertainty. Hence, for this reference object, with form deviations in the order of only 200 nm, the CMM is not able to reconstruct the profile. In case the form deviations of the measured object are in the same order of magnitude as the CMM errors, then it would probably be possible to detect a combined effect of the form deviations and CMM errors. When the form deviations are significantly larger compared to the CMM errors, then the rotated CMM measurements should be able to detect the profile of the object.

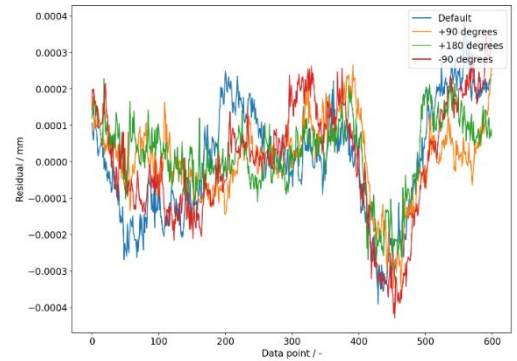


Figure 5: Residuals with respect to reference circle for the rotation measurements.

D. Bias correction

The mean diameter of the simulated MC samples is approximately equal to the diameter estimate based on the measurement data, so the diameter estimate is not biased. The mean roundness of the simulated MC samples is almost always higher than the roundness estimate of the measurement data, so the roundness estimate is biased. Therefore, the bias is subtracted from the roundness estimates for each measurement. The resulting E_n values can be found in Figure 6. The E_n values are all between -1 and 1, indicating consistency with the reference measurement. This provides a strong indication that the accuracy is improved after the bias correction. Applying the bias correction could result in a negative roundness estimate, which is physically impossible. It is most likely that the bias is overestimated due to an overestimated uncertainty of the error sources. In such a case, it is not recommended to correct the bias, but the high uncertainty will likely result in a sufficiently wide confidence interval, such that it captures the true roundness.

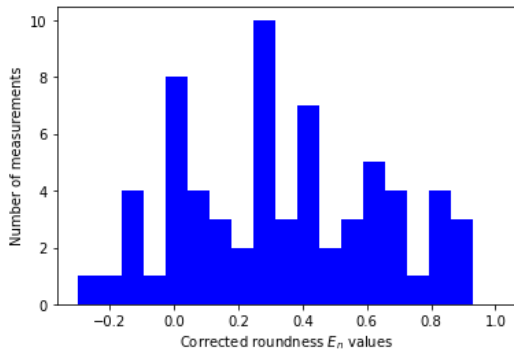


Figure 6: E_n values of the corrected roundness.

VI. CONCLUSION

In this research a digital model of a CMM was validated. These types of models are a crucial component in the digitalization of metrology and can only be properly used when validated. For different measurement locations and object orientations, the digital model produces outcomes are consistent with the reference. However, the roundness estimates seem to contain a bias. Assuming that the digital model is representative of the CMM, this bias can be estimated and corrected for to obtain more accurate results. Moreover, it was shown that the measurement location has a significant impact on the errors in the measurement data, while the orientation of the object had no significant impact in this study. However, the object orientation may become more relevant when the form deviations in the object are larger.

In addition to the work done in this research, it is also possible to correct for some of the CMM errors. This way, the estimated value of the measurand can be improved and

the uncertainty of the estimate reduced. Finally, it is important to note that there is added value in validating a digital model in multiple ways. For example, the digital model can also be validated with different measurement objects. This may expose different aspects of the digital model, i.e. an object with sharp edges may introduce additional errors to the data that are not covered by studying a smooth ring. Hence, the validation of the digital model is only relevant if the objects that are measured have similar features as the object that was used for the validation. If this is not the case, it is advisable to repeat the validation procedure with a different object. In addition, it is also advised to validate the digital model by means of a comparison with other parties, in order to confirm consistency with the outside world.

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